HOMEWORK 9
MATH 430, SPRING 2014

Problem 1. *Prove the Chinese Remainder Theorem*

Recall that in class we defined the formulas ϕ_{exp} and $\phi^*_{prime}(x,n)$, so that

(1) $\mathfrak{A} \models \phi_{exp}(e,n,k)$ iff $e^n = k$,
(2) $\mathfrak{A} \models \phi^*_{prime}(x,n)$ iff p is the n-th prime i.e. $p_n = p$.

Problem 2. *Show that* ϕ_{exp} *is* Δ_1 *by writing a formula in* Π_1 *form and proving that it is equivalent to* ϕ_{exp}.

Problem 3. *Show that* $\phi^*_{prime}(x,n)$ *is* Δ_1 *by writing a formula in* Π_1 *form and proving that it is equivalent to* $\phi^*_{prime}(x,n)$.

Problem 4. *Show that any model* $\mathfrak{B}$ *of* PA *is an end-extension of the standard model* $\mathfrak{A} := (\mathbb{N}, 0, <, S, +, \times)$. *I.e. show that there is a one-to-one function* $h : |\mathfrak{A}| \to |\mathfrak{B}|$, *such that:*

(1) *h is an isomorphism between $\mathfrak{A}$ and $range(h)$ (for the definition see the last problem of Hwk 8)and*
(2) *for every $b, c \in |\mathfrak{B}|$, if $b <_{\mathfrak{B}} c$ and $c \in range(h)$, then $b \in range(h)$.*